The Art of Destroying Flexagons

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Flexagons are (usually) flat models, often made of paper, that can be folded in certain ways to reveal additional faces besides the two that were originally the back and front of the model. Flexing is the word used to describe the special paper folding manipulation that has to be done in order to reveal a new face. Flexagons were invented and studied by Arthur Stone and some prominent fellow math graduate students from Princeton University in 1939, popularised by Martin Gardner in 1956[1] and again in 1988[2], and since studied by many recreational mathematicians (c.f. references in Wikipedia[3]). Once constructed, flexagons are beautiful objects, especially when their faces are colored, and flexing them is real fun and leads to some very interesting structures.

Surprisingly enough, even destroying flexagons can be fun *and* artistic! We suggest 4 different ways to artistically destroy flexagons, each with its own merit. Our exchange gift is a set of 4 flexagons strips, one for each demonstration.

1. Cutting through the center of flexagon leafs (recommended strip: tri-hexa-flexagon).

Fold a tri-hexa-flexagon. Then, starting from the outside hinge of a 2-leaf pat, make a cut through the center of the leaves, traversing all the leaves until you return to the initial cutting point. The result is a trefoil knotted, 8 times half-folded Möbius strip. This is because the tri-hexa-flexagon is analogous to a 3 half-twisted Möbius strip[4].

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[1] Gardner, Martin (December 1956). "Flexagons". *Scientific American*. Vol. 195 no. 6. pp. 162–168

[2] Gardner, Martin (1988). Hexaflexagons and Other Mathematical Diversions: The First Scientific American Book of Puzzles and Games. University of Chicago Press.

[3] https://en.wikipedia.org/wiki/Flexagon#Bibliography

[4] Elran, Y., & Schwartz, A. (2019). 16. Should We Call Them Flexa-Bands? In *The Mathematics of Various Entertaining Subjects* (pp. 249–261). Princeton University Press.
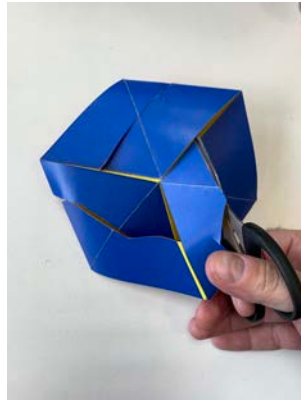
2. Cutting the corners and edges of flexagon leafs (recommended strip: hexa-hexa-flexagon).

Fold a hexa-hexa-flexagon. Then, stack all the leaves into one stack. The stack should be 18 leaves thick. Cut off each of the three corners of the flexagon. After cutting each corner it is recommended to open the flexagon and see the patterns that you get. We could call these "holy flexagons". Cutting off the corners of flexagons by just clipping the triangles' vertices is recommended as it eases flexing later on. The resulting flexagon can be further artistically enhanced by cutting patterns along the flexagons edges as well
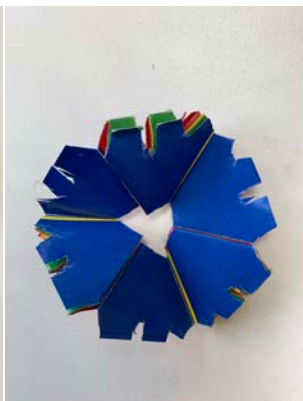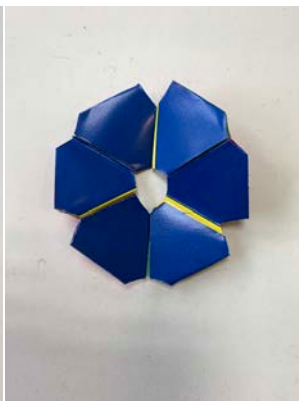
3. A circular cut in a flexagon (recommended strip: tri-hexa-flexagon).

Fold a tri-hexa-flexagon. Then, stack all the leaves into one stack. The stack should be 9 leaves thick. Make a circular cut, completely removing two of the flexagon's corners. Open up the result - it is a "flexa-frame"!
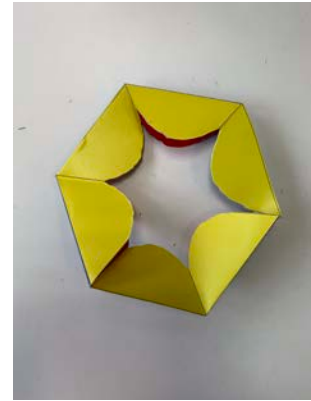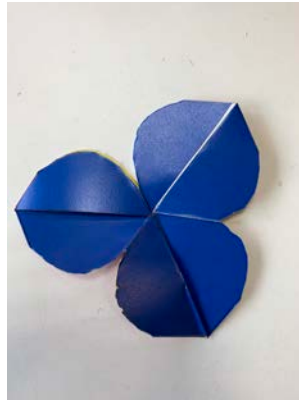
4. Cutting the flexagon before folding it (recommended strip: truncated hexa-hexa-flexagon).

Fold a hexa-hexa-flexagon from the truncated hexa-hexa-flexagon strip. Make sure that you still fold along the lines, even though (obviously) there will now be overlaps between different coloured leaves. The resulting flexagon can be flexed as usual, but the colourful patterns are very beautiful and provide insight into the inner structure of flexagons.