Stamping a Checker Board

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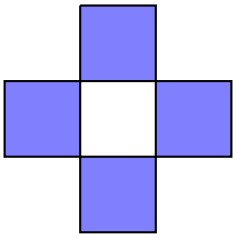
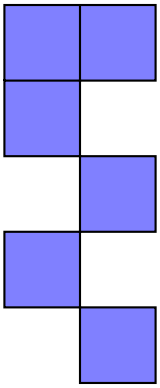
Suppose you have an $n \times n$ checker board with n odd. A *stamp with m blocks* is a configuration of m blocks that we do not require to be adjacent. For example, the following is a stamp with 4 blocks.

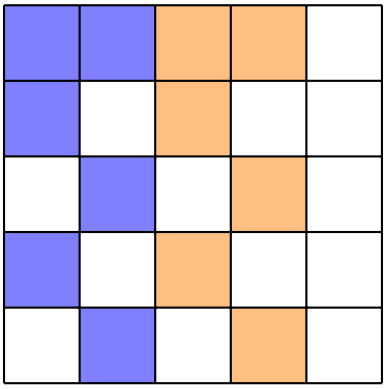


Can you use a stamp with $n+1$ blocks $n-1$ times to cover all but one block of your $n \times n$ checker board? The answer is yes! There is a way to cover the board such that the missing block is in the corner. Since n is odd, there exists an integer k such that $n = 2k+1$. If k is even, there is a way to cover the board so that the missed block is in the center. I will describe both ways of stamping the checker board in this brief article.

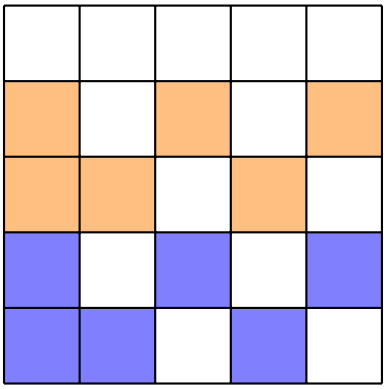
In order to cover all but one block of the board, the stamp with $n+1$ block must not hang off the board for any of the $n-1$ uses. For the covering that misses a block in a corner, consider a stamp on a $2 \times n$ grid where there is a block in the first column of the first row, a block in the first column of every even row, and a block in the second column of every odd row. Here is an example of the stamp when $n = 5$ to demonstrate.



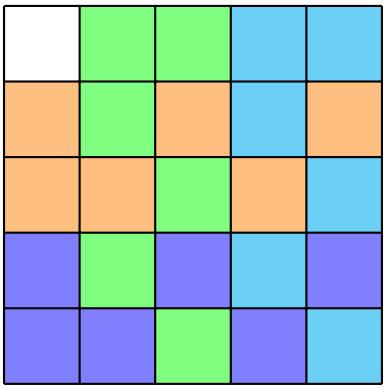
Since there is one block in rows 2 to $n-1$ and two blocks on the first row, there are a total of $n+1$ blocks in the stamp. Starting at the top left corner of the board, use the stamp k times as you move to the right. The result should have all but the last block of the first row of the board covered and a checkered covering of rows 2 to n and columns 1 to $n-1$. This is depicted on a 5×5 board in the image below.



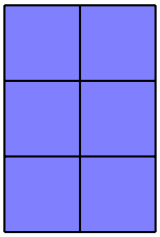
Now turn the board a quarter turn counterclockwise. The top row should now have no blocks covered.



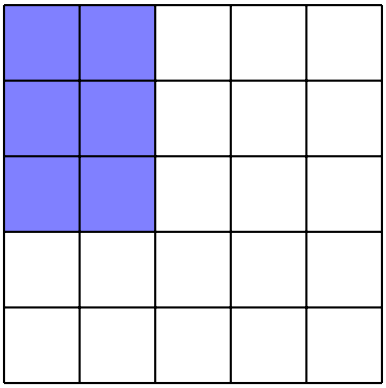
Starting at the second column, use the stamp k times as you moved to the right. This will fill in the checkered part of the board from row 2 to n and column 2 to n and all but the first block of the first row. The full stamp covering is demonstrated on a 5×5 board below.



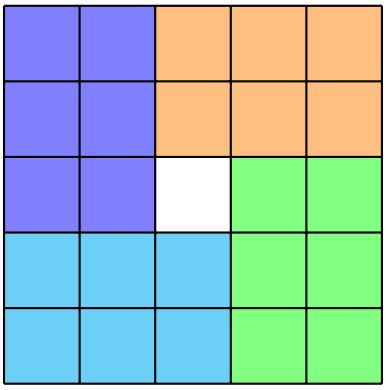
For the covering that misses the center block, suppose that $n = 2k + 1$ with k even. Consider a stamp that is 2 blocks by $k + 1$ blocks. Then the stamp has $2(k + 1) = 2k + 2 = n + 1$ blocks. The stamp for a 5×5 grid is shown below.



Starting at the top left corner of the board, stamp the board $k/2$ times. Since k is even, this is an integer. The result is illustrated below for a 5×5 board.



Now turn the board a quarter turn counterclockwise. Starting at the top left corner of the board, stamp the board $k/2$ times. Repeat this action 2 more times, and the board will be covered with only the center block not stamped. The full stamp covering is demonstrated on a 5×5 board below.



This problem was a generalization of a question on the *Fall 2019 Tournament of Towns*.