Multiplication by Superposition

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Multiplying polygons by superposition, Mps

For G4G14 this is an idea that can be used to devise various puzzles. The idea is to overlap two geometric figures and count the number of pieces produced if all lines are cut thru.   The simplest case is two equal circles. We can overlap them in three different ways to get a^2' = 1, 2 and 3 pieces as shown in Fig 1. Call this multiplication Mps. These counts are the complete solution for this Mps. The primed expression, c^2', indicates Mps here.
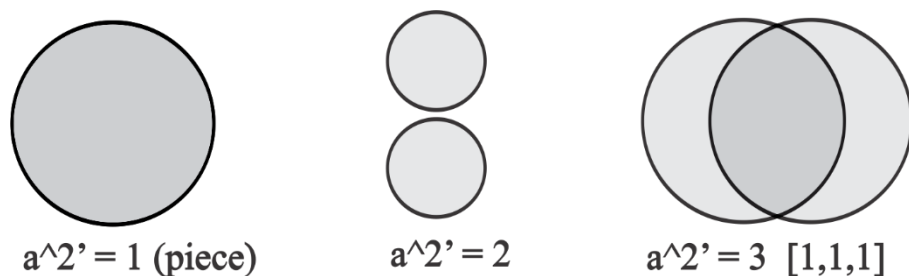
a^2' = 1 (piece)   a^2' = 2   a^2' = 3 [1,1,1]

Figure 1

Figure 2 shows Mps solutions for 2 equal triangles a with a^2' having 7 solutions, 1 thru 7 pieces.

a^2' = 1   a^2' = 2   a^2' = 3 [1,1,1]   a^2' = 4 [2,1,1]

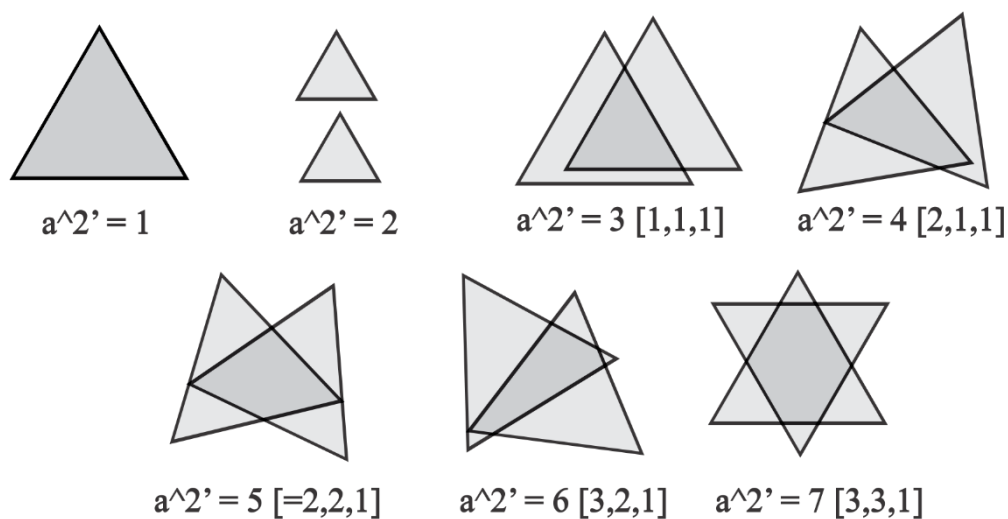a^2' = 5 [=2,2,1]   a^2' = 6 [3,2,1]   a^2' = 7 [3,3,1]

Figure 2

Continuing the sequence with a square we get 9 Mps solutions in Figure 3, and 11 solutions with a pentagon in Figure 4. The arrows in the 10 piece solution show where tiny pieces are located
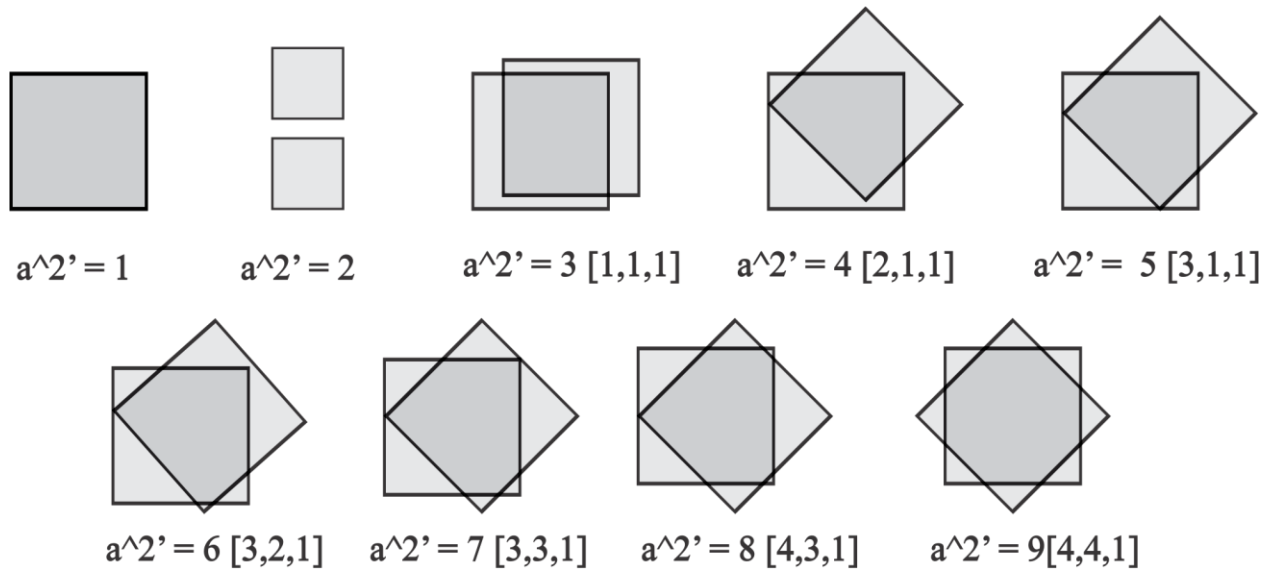
a^2' = 1   a^2' = 2   a^2' = 3 [1,1,1]   a^2' = 4 [2,1,1]   a^2' = 5 [3,1,1]

a^2' = 6 [3,2,1]   a^2' = 7 [3,3,1]   a^2' = 8 [4,3,1]   a^2' = 9[4,4,1]

Figure 3

a^2' = 1   a^2' = 2   a^2' = 3 [1,1,1]   a^2' = 4 [2,1,1]   a^2' = 5 [2,2,1]   a^2' = 6 [3,2,1]

a^2' = 7 [3,3,1]   a^2' = 8 [4,4,1]   a^2' = 9 [4,4,1]   a^2' = 10 [5,4,1]   a^2' = 11 [5,5,1]

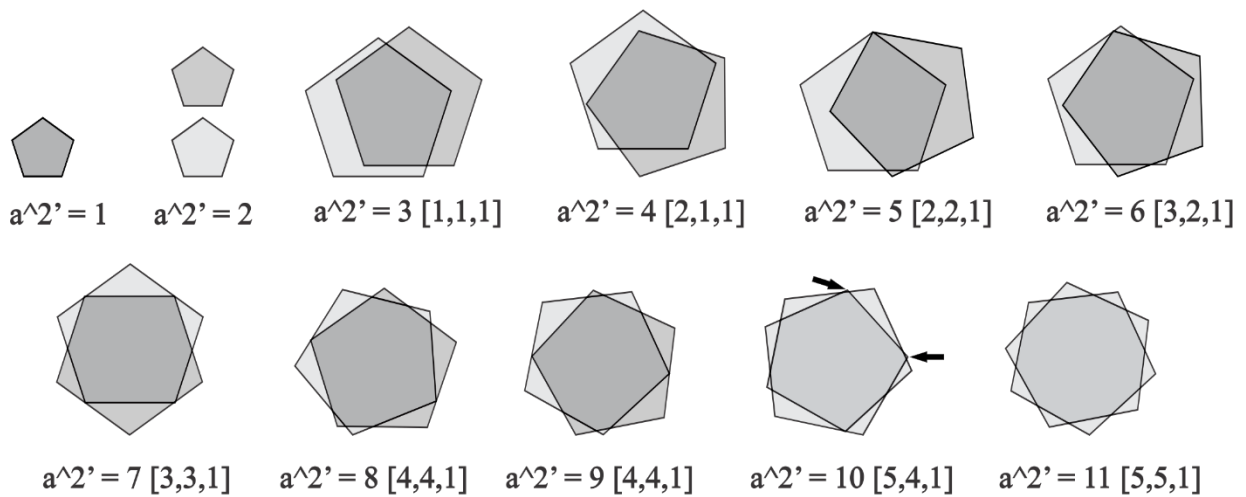Figure 4

The number of solutions shown here for a regular polygon with n sides is 2n+1 for Mps. This has not been proven but is a conjecture based on the three solutions, triangle, square, pentagon. This could easily be refuted if the hexagon does not permit some solution less than 13.

If the a^2' total solutions = 1 thru n+1 pieces continues for regular polygons the solutions will have increasing numbers of smaller pieces as n increases with a few large pieces as seen in the figures.

As n gets really large but still finite the polygons get more and more circular yet the number of solutions are n+1 'large'. At infinity we have a circle with only 3 solutions. Perhaps instead of solutions increasing they start to decrease as a ratio of s/n.

Spin Multiplication by superposition

Spin Multiplication of squares by superposition Sms

A square can be overlapped on top of itself and rotated 45 degrees to get 9 pieces when all lines are cut thru as shown in Figure 5. We overlap this product on itself and rotate it 22.5 degrees to get the next product having 49 pieces and then 11.25 degrees to get 225 pieces.
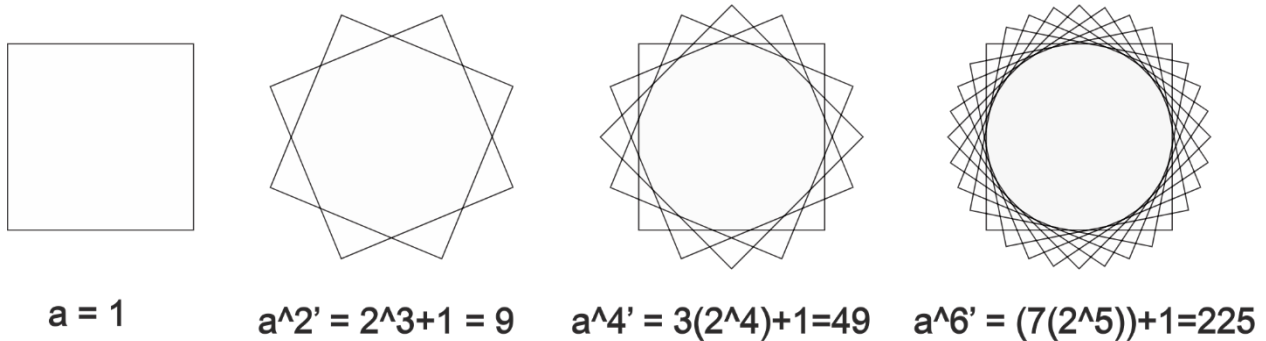

a = 1   a^2' = 2^3+1 = 9   a^4' = 3(2^4)+1=49   a^6' = (7(2^5))+1=225

Figure 5

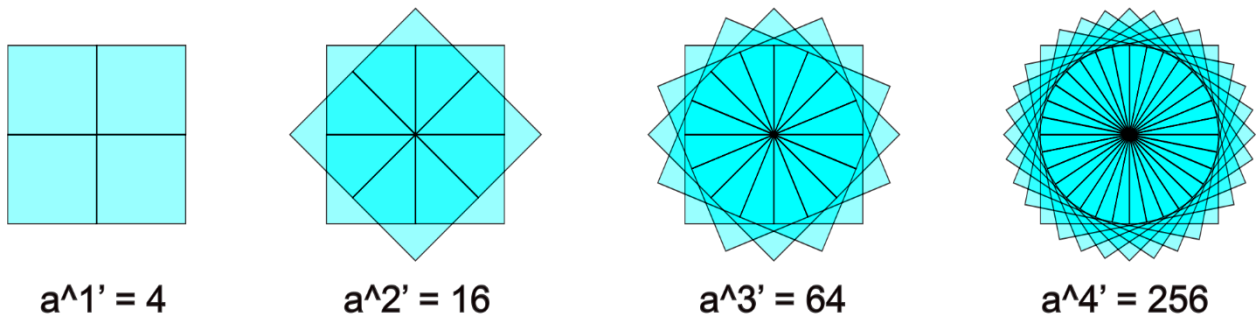Overlapping a 2x2 grid gives the exponential sequence shown in Figure 6.

| a^1' = 4 | a^2' = 16 | a^3' = 64 | a^4' = 256 |

Figure 6

Odd order square grids have a slewed exponential sequence of pieces as seen in Figure 7 with a 3x3 grid.

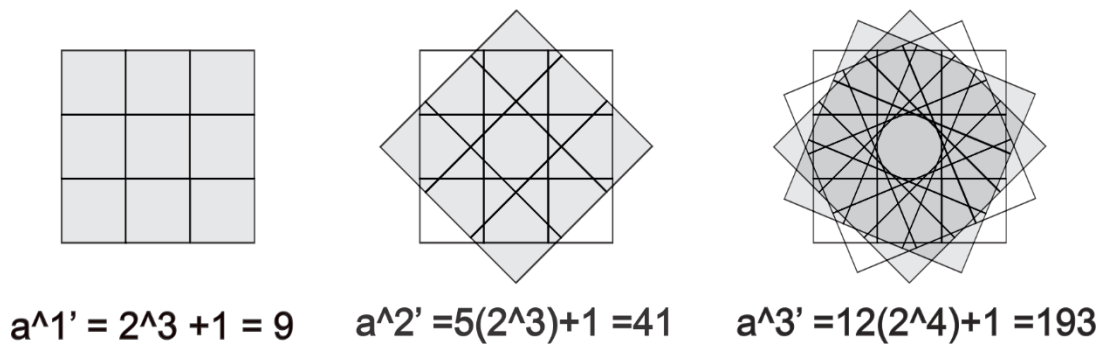a^1' = 2^3 +1 = 9   a^2' =5(2^3)+1 =41   a^3' =12(2^4)+1 =193

Figure 7

Figure 8 with a 4x4 grid being even order returns a simple exponential sequence.
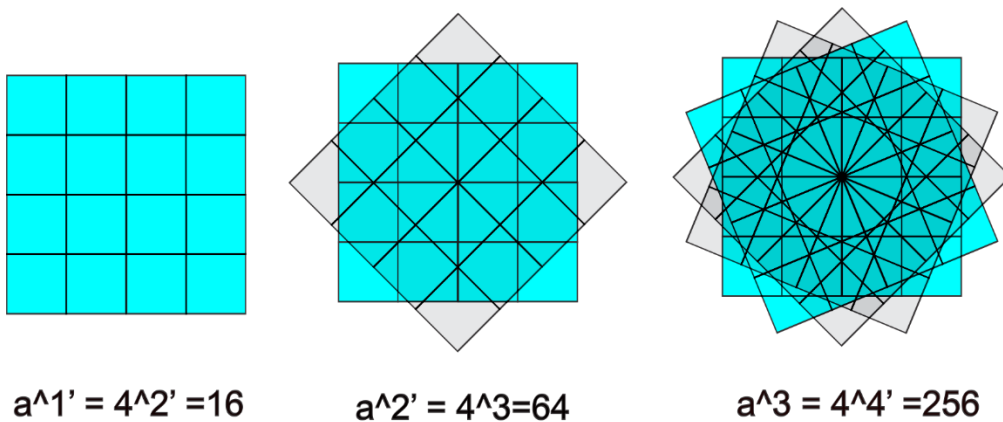
a^1' = 4^2' =16   a^2' = 4^3=64   a^3 = 4^4' =256

Figure 8

This regularity continues for all sorts of regular grids under this kind of binary symmetrical spin multiplication.

$a^1{}' = 1$ $a^2{}' = 1x6+1 = 7$ $a^4{}' = 3x12+1 = 37$ $a^6{}' = 7X24+1 = 169$

$a^1{}' = 4$ $a^2{}' = 3x6 + 1 = 19$ $a^4{}' = 7x12 + 1 = 85$ $a^6{}' = 15x24 + 1 = 360$
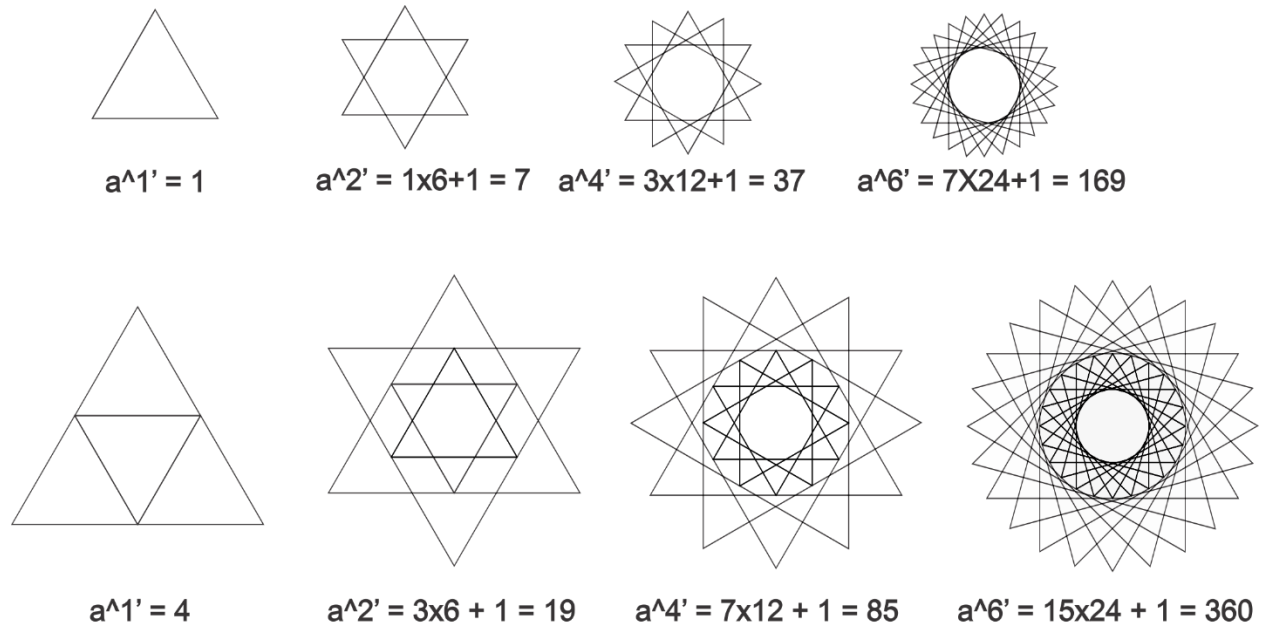
Figure 9

Things get more complicated if we allow all three operations, translation rotation and spin.   This was attempted in Figure 10 with results as shown.

a = 4   6   7   8   9   10   11
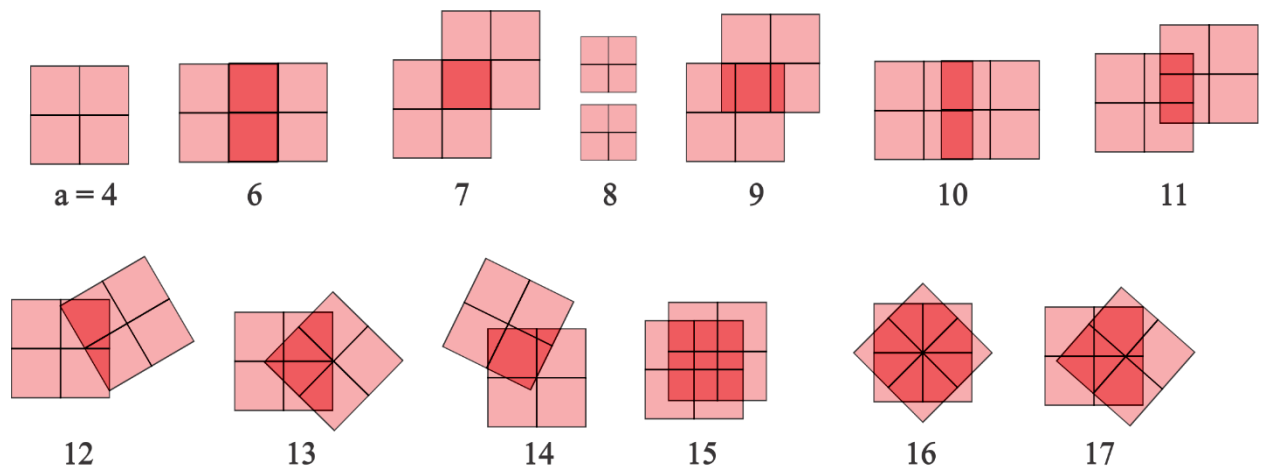
12   13   14   15   16   17

Figure 10

There is much more to explore